

The classical theory of brittle fracture, the Griffith theory (Griffith, 1921, 1925), was originally developed to account for the discrepancy between the theoretical and observed strengths of materials. Griffith postulated that solids contain many small cracks, which he represented, in his two-dimensional model, by extremely eccentric elliptical holes. He demonstrated that the stress near the ends of certain holes, of critical length and orientation, may reach the theoretical value when the stress in the material as a whole is considerably below this value. These cracks will then be propagated and lead to failure of the material. Griffith (1925) derived a fracture condition for biaxial states of stress, assuming a mechanically isotropic material containing elliptical cracks of random orientation, the maximum crack-lengths and radii of curvature of the ends of cracks being the same for cracks of all orientations. Failure is predicted when the highest tensile stress at the ends of the longest crack of the most dangerous orientation reaches the value of the theoretical strength of the material. In compression tests the cracks that will propagate and lead to failure are inclined at 45° to the greatest and least principal (compressive) stresses, σ_1 and σ_3 respectively. Fracture should occur when

$$(1) \quad \sigma_3 = -K, \quad \text{if } \sigma_1 + 3\sigma_3 < 0, \text{ and}$$

$$(2) \quad (\sigma_1 - \sigma_3)^2 - 8K(\sigma_1 + \sigma_3) = 0, \text{ if } \sigma_1 + 3\sigma_3 > 0.$$

The constant K in these equations represents the tensile strength for a uniaxial tensile stress ($\sigma_3 < 0, \sigma_1 = 0$). It has been suggested (Orowan, 1949) that the condition represented by equations (1) and (2) will hold for cases where the stress is triaxial, since normal and shear stresses acting normal to the plane of σ_1 and σ_3 cannot appreciably affect the stresses in this plane. Orowan (1949, p. 201) has shown that the fracture strengths of some brittle materials at high confining pressures can be correlated satisfactorily with their tensile strengths, experimentally determined, using the Griffith fracture criterion. Although the assumptions of mechanical isotropy and, perhaps, random orientation of cracks are not strictly valid for quartz, it is of interest to determine whether the Griffith fracture condition gives a good correlation between the strengths obtained in the present series of experiments and those determined under other conditions.

Since the average strength of crystals compressed parallel to the c -axis at 27 kb confining pressure is 46.5 kb, σ_1 and σ_3 at failure are 73.5 kb and 27 kb, respectively. Substituting these values in equation (2) above, a value of the constant K , equal to 2.69 kb, is obtained. This should be the tensile strength of a crystal extended parallel to the c -axis. The maximum tensile strength at one atmosphere pressure recorded by Berndt (Sosman, 1927) in thirteen tests was 1.19 kb and the average was 1.1 kb, which are approximately half the value of K . In view of the nature of these tests and the methods by which samples are generally prepared, (by sawing, grinding, or core drilling) the likelihood of surface flaws is high, and measured strengths are likely to be low. Using the value of K obtained above, the strengths ($\sigma_1 - \sigma_3$) at different confining pressures (σ_3) were calculated using the Griffith equation (2). These are represented by the curve in figure 9a. The strengths recorded by Berndt, Goranson (1949) and Griggs, Turner, and Heard (1960), at one bar confining pressure,

Griggs and Bell (1938) at 8 to 15 kb, and Bridgman (1941) at 15 to 25 kb lie quite close to this curve. On the other hand, values of strength obtained by Griggs, Turner, and Heard (1960) at 2.5 to 5 kb and some of those obtained by Bridgman (1940), Goranson (1949), and Griggs and Bell (1938) at higher confining pressures are considerably higher than the strengths indicated by the curve (fig. 9a). For reasons noted above, these latter measurements may be higher than the true values.

The experimental data for cylinders compressed perpendicular to r and m are represented in figure 9b, along with the curve of the Griffith equation, calculated using values of K obtained from our strength determinations. The two values of K are almost identical (K is 2.35 and 2.36 kb for r and m cylinders, respectively), and the values of $(\sigma_1 - \sigma_3)$ for both orientations are represented by a single curve. Few data are available on the strength for these orientations at other confining pressures.

No definite conclusions can be drawn as to the consistency of the data with the predictions of the Griffith theory of brittle fracture. But many of the earlier strength measurements, particularly those made in compression tests at one atmosphere, which are not likely to involve such large errors as measurements at high confining pressures, can be satisfactorily correlated with the measurements made in the present study by means of the Griffith fracture criterion.

It was demonstrated above that faulting takes place at lower shear stress on the base than on r and z and on r and z than on m and a . It was shown that this is not accounted for by the bond density across these planes, as in cleavage development, or by the elastic moduli. Nor is it in accord with the theoretical strengths for slip in these planes in the direction of the shear displacements on the faults. This is not unusual, as the plasticity of crystals is not generally related to their behavior below the elastic limit. To determine if the shear strengths along these planes are limited by plastic yielding, we should therefore consider the actual plastic behavior of quartz, insofar as this is known. Large plastic deformations of quartzite and quartz single crystals were recently obtained for the first time in the laboratory of D. T. Griggs (Carter, Christie, and Griggs, 1961; Christie, Carter, and Griggs, in press). In short tests quartz flows plastically at high confining pressures (above 15 kb) and moderate to high temperatures (300° to 1500°C) and also at lower pressure (5 kb) and moderate temperatures (300° to 500°C) in tests at slow strain-rates (Heard, 1962). Under these conditions slip on the base (c) is the commonest mechanism, but other mechanisms also operate; there is evidence that the other slip-planes include the rhombohedra r and z , and possibly also the prism planes. The critical resolved shear stress for basal slip is lower than for the other mechanisms. It is therefore possible to account for the greater ease of faulting on the base than on the other planes if it is postulated that faulting in our experiments is initiated by small amounts of slip on the planes.

The hypothesis that the faulting is initiated by small amounts of plastic flow is not inconsistent with the use of the term "brittle fracture" for the phenomenon. The term "brittle fracture" specifies only the mode of propagation of the existing cracks in a stressed body: it denotes that the only work done in crack propagation is that required to overcome the cohesive forces between